Wind induced actions and action effects on slender structures with special emphasis on wind climate modelling

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Abstract
It is much important to fit the cyclone of Bay of Bengal in to suitable statistical model so that the specification of design wind speed due to cyclone for a particular return period can be estimated. The wind load is proportional to the squire of wind velocity so the analysis of wind climate is important for the specification of design wind speed and hence design wind load. The analysis of extreme wind climate is required for along wind vibration which is mainly due to gust induced vibration & the analysis of general wind climate is required for cross wind vibration which is mainly due to periodic vortex shedding.
In the least square method, the values of s, k and threshold value are varied over a wide range and the sum of the square of the between the observed and theoretical value is calculated for each case. Then those particular values of s and k will be selected for which the sum of the square of error will be minimum. The quality of fitting for this case should also be checked. The shape and scale parameters in GPD (Generalized Pareto Distribution) have been found k = -0.11, s = 13.86m/s (by Method of moment) & k = -0.15, s = 14.53m/s (by Least Square method) corresponding to least summation of squares of errors. It has also observed that corresponding least summation of squares of errors the threshold value is 20.0 m/s.

Introduction
Tall and slender structures are flexible and exhibit a dynamic response to wind. Tall structures vibrate in wind due to the turbulence inherent in the wind as well as that generated by the structure itself due to separation of the flow. In case of tall structures, the dynamic forces acting in the direction of wind flow give the along wind response of the structure. However, the forces can act in a direction nearly perpendicular to the wind flow, exhibiting an across-wind response.
There are mainly 3 types of natural hazards, namely,

(a) Natural hazards due to extreme snow events
(b) Natural hazards due to strong earthquakes
(c) Natural hazards due to extreme winds

High wind speed storms and cyclones occur in many parts of India. Every year a lot of structures in coastal regions of India especially those which are in the east coast of India are affected by extreme winds. Some cyclone prone coastal states are Gujarat, Tamilnadu, Andhra, Orissa and West Bengal.
Wind Storms fall into various types: (1) Frontal depressions, (2) cyclone/Hurricanes/Typhoons, (3) tornados, (4) Thunderstorms, (5) Squalls, (6) Dust devils, (7) Down slope winds (8) Monsoon gales.
Amongst all these types of storms, cyclones constitute the worst threat of causing disasters. It is seen that Indian regions gets less than 10% of events that occur globally, i.e., about 6 cyclones per year, mostly on east coast. Yet the Indian sub-continental region has been suffered by most of cyclone disasters that occurred. **[1]**
So this paper consists the cyclone modeling of Bay of Bengal which is a high cyclone prone area of east coast region of the India. The wind load is proportional to the square of the wind speed (V). In extreme wind conditions, the gustiness from the oncoming flow will cause gust induced vibration which will lead to forced rupture. So the specification of design wind speeds are prerequisites for the specification of design wind loads. The extreme wind climate modelling is required for this purpose.

The aim of the work is to fit the non exceedance probability of a cyclone event per year in a suitable probability distribution. From the basic probability theory it can be easily proved that the non exceedance probability of wind speed per year can be given by the relation:

$$p(V \leq V_{ref})_{year} = \sum_N p(N).p_{event}(V \leq V_{ref})^N \quad , \quad (1)$$

Where p(N) = probability of N storms per year determined by Poisson distribution and p_{event} = probability of $V \leq V_{ref}$ per event, where V_{ref} is the reference velocity, p_{event} can be fitted by Generalized Pareto Distribution (GPD).

As storm events are rare events, the probability of N storms per year, p(N) is low. So p(N) can be attempted to fit by Poisson's distribution. On the other hand the probability of the event, p_{event} can be analysed by 'peaks over threshold' approach **[2]**. This approach enables the analyst to use all the data exceeding a sufficiently high threshold, and is more effective than the classical approach, which uses only the largest value in each of a number of basic comparable sets **[2]**. So p_{event} can be fitted by Generalised Pareto distribution. A.C.Davison and R.L. Smith **[3]** mentioned that if p(N) follows Poisson distribution and p_{event} follows GPD, then the non-exceedance probability per year, $p(V \leq V_{ref})_{year}$ will follow generalised extreme value distribution. So $p(V \leq V_{ref})_{year}$ can be fitted either by Type I (Gumbel), are different types Type II (Fřechet), or Type III (Reverse Webull distribution) distributions with different curvatures parameter.

The non-exceedance probability per year for other storm phenomena of India can also be found in a similar way and can be fitted with the appropriate extreme value limit distribution. For this purpose the individual ensembles for each of the different storm phenomena of India will be sampled for a large number of stations and the combination of probability densities from different storm types will be done. These works will be aimed to develop the specification of design wind speed. Cyclones affect both the Bay of Bengal and the Arabian Sea. They are rare in Bay of Bengal from January to March. Isolated ones forming in the South Bay of Bengal move west north westwards and hit Tamil Nadu and Sri Lanka coasts. In April and May, these form in the South and adjoining Central Bay and move initially northwest, north and then recurve to the northeast striking the Arakan coasts in April and Andhra-Orissa-West Bengal-Bangla Desh coasts in May. Most of the monsoon (June - September) storms develop in the central and in the North Bay and move west-north-westwards affecting Andhra-Orissa-West Bengal coasts. Post monsoon (October-December) storms form mostly in the south and the central Bay, recurve between 15^{0} and 18^{O} N affecting Tamil Nadu-Andhra Orissa-West Bengal-Bangla Desh coasts. This is a yearly extreme. [4]

Statistical Methodologies:

The main steps of statistical methodologies, used in this scope of study, are pointed as below:

1. The hourly mean wind speed data for east coast region of India (i.e. Bay of Bengal) have been supplied by Indian Meteorological Department, Pune. Since cyclones are characterised by maximum sustained surface wind speed. For this purpose sustainable surface wind speed should be taken which are take greater than the chosen threshold value.

2. The threshold values of the cyclones are varied in order to determine the appropriate threshold value. The order statistics can be used at this stage. In this approach the surface wind speed data with ensemble size N are sorted in ascending order. From this sorted list, the relative frequency ($f_{relative}$) of non-exceedence of the gust wind speed (x_i) can be calculated using the relation below: **[5]**

$$f_{relative}(x \leq x_i) = \left(\frac{i-\alpha}{N+\beta}\right) \quad (2)$$

i – Rank in list of ascending order
N - Ensemble size
x_i – Gust wind speed(m/s)

The parameters α & β define the 'plotting position' for the reduced variate. α & β are related by the equation:

$$2*\alpha+\beta=1$$

3. For acceptance tests with Gumbel probability paper, the recommended values of α & β are 0.44 and 0.12. The analogous numbers for the type III distribution are 0.30 and 0.40. **[8]**

The above relative frequencies verses gust wind speeds are plotted in Gumbel probability paper. The curve obtained must be fitted by the Generalized Pareto distribution. The Generalized Pareto distribution is given by **:**

$$p(v \leq v_{ref}) = 1-\left[1-\frac{v_{ref}-v_s}{s}.k\right]^{1/k} \quad [6] \quad (3)$$

p – Non exceedance probability
v_{ref} - Gust wind speed (m/s)

v_s- Threshold value (m/s)
s- Scale parameter (m/s)
k- Shape parameter
4. The method of moments can be used to fit the non-exceedance probability by Generalised Pareto distribution. Based on the method of moments, the two parameters, s and k, are obtained from the sample mean and sample standard deviation as follows:

$$s = 0.5.v_{mean}.\left(1+\left(\frac{v_{mean}}{v_{sdev}}\right)^2\right) \quad , \quad k = 0.5\left(1-\left(\frac{v_{mean}}{v_{sdev}}\right)^2\right) \tag{4}$$

v_{mean} and v_{sdev} have to be calculated for the exceedance of v_s i.e.,

$$v_{mean} = \frac{1}{N}\sum_{i=1}^{N}(v_i - v_s) \tag{5}$$

$$v_{sdev} = \left(\frac{1}{N-1}\sum_{i=1}^{N}(v_i - v_s - v_{mean})^2\right)^{1/2} \tag{6}$$

$p(V \le V_{ref})_{year}$

will follow generalised extreme value distribution. So $p(V \le V_{ref})_{year}$ can be fitted by either by Type-I (Gumbel), are different types Type-II (Fŕechet), or Type-III (Reverse Weibull distribution) distributions. Actual probability should be determined by the expression given below:

$$p(V \le V_{ref})_{year} = \sum_{i=1}^{n} p(i) * p_{event}{}^{i} \tag{7}$$

Where $p(i)$ = Observed probability of 1, 2, 3 & 4 cyclones per year.

$p_{event}{}^{i}$ = Observed probability of individual cyclones obtained from order statistics.

For finding the best estimator of gust wind speed distribution should be fitted the wind speed data into Gumbel distribution (Type I), Fŕechet distribution (Type II) and Reverse Weibull distribution (Type III).

Determination of Gumbel & Reverse Weibull parameters:

In Type II & Type III distribution there is a variables f_1 , f_2 , m & σ . Where f_1 & f_2 are given expressions as:

$$f_1 = \Gamma(1+\tau) \quad \& \quad f_2 = \sqrt{\Gamma(1+2\tau) - f_1{}^2} \tag{8}$$

The Method of Least Square is a procedure to determine the best fit line to data; The basic problem is to find the best fit straight line y=ax+ b given that for n ε {1,....N}, so Linearised Type II & Type III distribution in form of $y = ax + b$ an estimated the all values of y corresponding to each Gust wind speed (x) and find out the value of a & b by the Least square methods. Where the expressions of a and b given below:

$$b = \left(\frac{\sum_{i=1}^{n}(x_i)^2\sum_{i=1}^{n}y_i - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}x_i y_i}{n\sum_{i=1}^{n}(x_i)^2 - \left(\sum_{i=1}^{n}x_i\right)^2}\right) \tag{9}$$

$$a = \left(\frac{n\sum_{i=1}^{n}x_i y_i - \sum_{i=1}^{n}x_i\sum_{i=1}^{n}y_i}{n\sum_{i=1}^{n}(x_i)^2 - \left(\sum_{i=1}^{n}x_i\right)^2}\right) \tag{10}$$

The determined value of a and b should be used further for the determination of parameters m & σ by the expressions:If curvature parameters τ ≠ 0 expressions are:

$$\sigma = -\frac{f_2}{a} \quad , \quad m = -\frac{(b - f_1)}{a}$$

For curvature parameter τ = 0 expressions are:

$$\sigma = \frac{\Pi}{\sqrt{6}} * \frac{1}{a} \quad , \quad m = \frac{\gamma - b}{a}$$

Probability distribution for extreme events by using Generalized Pareto Distribution (GPD):

As a first step of the work Bay of Bengal has been chosen for the study. The results obtained after fitting the non-exceedance probabilities of surface wind speeds by Generalized Pareto distribution using the method of moment have been shown below in figure 3 without separating threshold values.

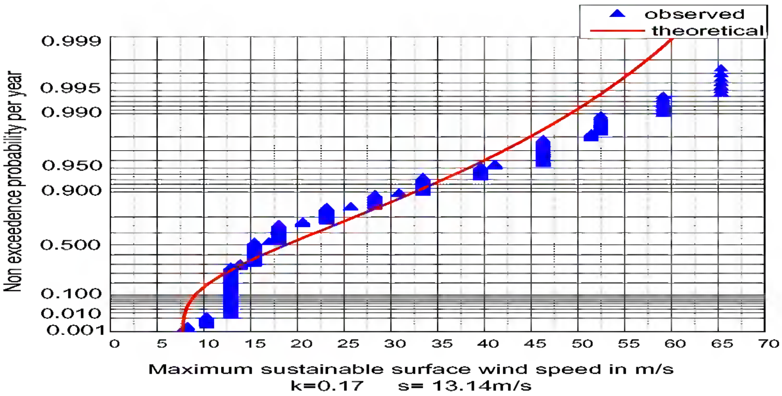

Fig. 1 Observed & Theoretical probability Vs Maximum sustainable surface wind speed in m/s

It has been observed from the above fig1.that there is enough deviation of the observed values from the theoretical distribution. So it has been seen that the quality of fitting of curve is not good. However, a decision table (Table 1) is given below to choose the appropriate threshold value based on the design wind speed corresponding to the non-exceedance probability of 0.999. There are more values of threshold value and corresponding shape and scale parameters etc. But shown only some values in a given below table1.

Table 1: Decision table for the choice of the appropriate threshold value by Method of Moment:

Sl.No.	Threshold Value (m/s)	Design wind Speed (m/s)	Shape parameters	Scale parameters (m/s)	Ensemble Size (N)
1	20.0	85.3	-0.11900	13.86553	513
2	20.1	86.3	-0.10905	13.63132	513
3	20.2	87.2	-0.09918	13.40009	513
4	20.3	88.2	-0.08939	13.17180	513
5	20.4	89.2	-0.07968	12.94644	513
6	20.5	90.2	-0.07006	12.72398	513
7	20.6	82.7	-0.15276	14.55950	479
8	20.7	83.6	-0.14246	14.31522	479
9	20.8	84.5	-0.13225	14.07401	479
10	20.9	85.5	-0.12211	13.83586	479
11	21.0	86.4	-0.11206	13.60074	479
12	21.1	87.4	-0.10210	13.36861	479
13	21.2	88.3	-0.09221	13.13947	479
14	21.3	89.3	-0.08241	12.91328	479
15	21.4	90.3	-0.07268	12.69002	479
16	21.5	91.3	-0.06304	12.46966	479
17	21.6	92.4	-0.05348	12.25219	479
18	21.7	93.4	-0.04401	12.03756	479
19	21.8	94.5	-0.03461	11.82577	479
20	21.9	95.5	-0.02530	11.61679	479
21	22.0	96.6	-0.01607	11.41059	479
22	22.1	97.7	-0.00692	11.20714	479
23	22.2	98.8	0.00215	11.00643	479
24	22.3	99.9	0.01114	10.80843	479
25	22.4	101.0	0.02004	10.61310	479
26	22.5	102.2	0.02887	10.42044	479
27	22.6	103.3	0.03761	10.23042	479
28	22.7	104.5	0.04626	10.04300	479
29	22.8	105.6	0.05484	9.85817	479
30	22.9	106.8	0.06334	9.67589	479
31	23.0	107.9	0.07175	9.49616	479
32	23.1	109.1	0.08008	9.31894	479

33	23.2	66.2	-0.49748	22.11135	326
34	23.3	66.7	-0.48401	21.76413	326
35	23.4	67.2	-0.47064	21.42094	326

It has been observed from the above table1.that there is no stability in the values of design wind speed corresponding to the non-exceedance probability of 0.999. This clearly indicates that the slight change in the threshold value gives the variation in design wind speed.

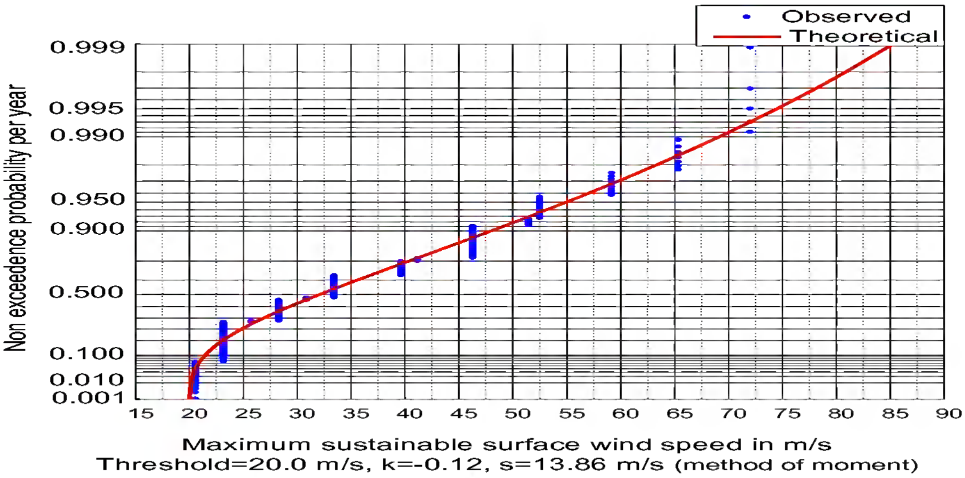

Fig. 2 Observed & Theoretical probability Vs Maximum sustainable surface wind speed in m/s

The results obtained after fitting the non-exceedance probabilities of surface wind speeds by Generalized Pareto distribution using the method of moment have been shown in above figure 2. It has been also observed that the quality of fitting of above curve is not good because there is slight deviation of the observed values from the theoretical distribution (using method of moment).

For fitting of the best curve an alternative method, least squire method is used. The result obtained by using this method for the same threshold value (20. m/s) have been shown in given below figure 3.

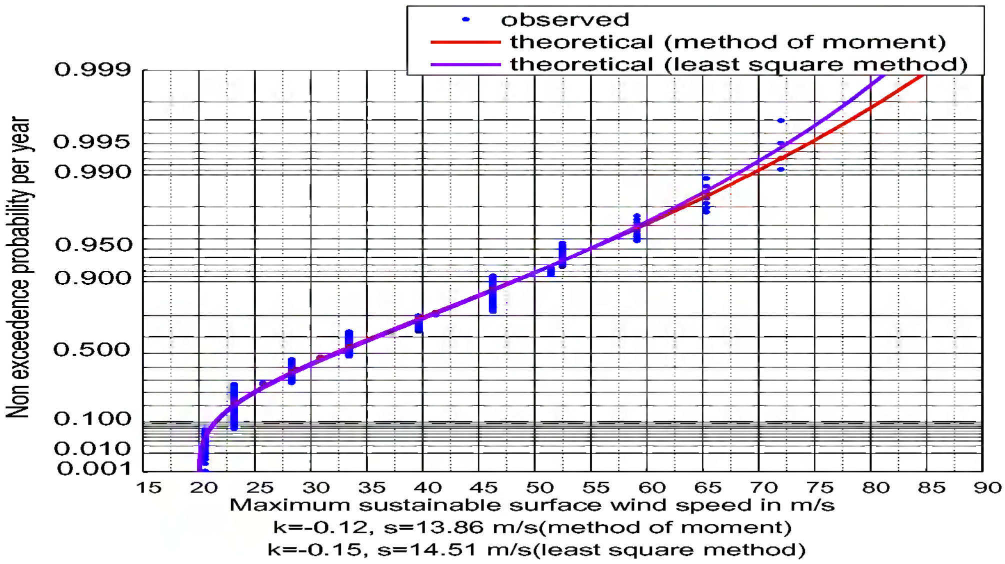

Fig. 3 Observed &Theoretical probability Vs Maximum sustainable surface wind speed in m/s

It can be observed from the above two figures (Figure 2,3) that the least square method is providing much better fit rather than the method of moment. Now for determining the appropriate threshold value the summation of the squares of the error of the observed value from the theoretical one should be calculated and the threshold value, for which this summation will be the least, should be chosen as the appropriate one. There are more values but some values have been illustrated in table 2.

Table 2: Decision table for the choice of the appropriate threshold value by Least Square Method:

Sl. No.	Threshold Value(m/s)	Design wind	Shape parameters	Scale parameters	Ensemble Size (N)	Error	Error/En-semble size(N)

		Speed (m/s)		(m/s)			
1	20.0	81.7	-0.15400	14.51553	513	35.133	0.0684
2	20.1	81.9	-0.14905	14.33132	513	35.175	0.0685
3	20.2	82.8	-0.13918	14.10009	513	35.563	0.0693
4	20.3	83.0	-0.13439	13.92180	513	36.502	0.0711
5	20.4	83.8	-0.12468	13.69644	513	38.566	0.0751
6	20.5	84.0	-0.12006	13.52398	513	44.053	0.0858
7	20.6	80.3	-0.18276	15.20950	479	47.846	0.0998
8	20.7	80.5	-0.17746	15.01522	479	47.680	0.0995
9	20.8	80.7	-0.17225	14.82401	479	47.619	0.0994
10	20.9	81.5	-0.16211	14.58586	479	47.650	0.0994
11	21.0	81.7	-0.15706	14.40074	479	47.805	0.0998
12	21.1	82.1	-0.15210	14.26861	479	48.093	0.1004
13	21.2	82.3	-0.14721	14.08947	479	48.527	0.1013
14	21.3	83.2	-0.13741	13.86328	479	49.122	0.1025
15	21.4	83.3	-0.13268	13.69002	479	49.904	0.1041
16	21.5	84.2	-0.12304	13.46966	479	50.897	0.1062
17	21.6	84.4	-0.11848	13.30219	479	52.133	0.1088
18	21.7	85.2	-0.10901	13.08756	479	53.641	0.1119
19	21.8	86.1	-0.09961	12.87577	479	55.477	0.1158
20	21.9	86.8	-0.09030	12.61679	479	57.683	0.1204
21	22.0	87.7	-0.08107	12.41059	479	60.337	0.1259
22	22.1	88.6	-0.07192	12.20714	479	63.529	0.1326
23	22.2	89.5	-0.06285	12.00643	479	67.377	0.1406
24	22.3	91.0	-0.04886	11.70843	479	72.035	0.1503
25	22.4	91.9	-0.03996	11.51310	479	77.722	0.1622
26	22.5	93.7	-0.02613	11.27044	479	84.757	0.1769
27	22.6	95.3	-0.01239	10.98042	479	93.623	0.1954
28	22.7	97.2	0.00126	10.74300	479	105.08	0.2193
29	22.8	99.2	0.01484	10.50817	479	120.55	0.2516
30	22.9	103.0	0.03834	10.12589	479	142.89	0.2983
31	23.0	105.2	0.05175	9.89616	479	179.69	0.3751
32	23.1	108.6	0.07008	9.61894	479	270.04	0.5637
33	23.2	75.0	-0.37748	21.11135	326	47.511	0.1457
34	23.3	75.2	-0.36901	20.76413	326	46.085	0.1413
35	23.4	75.3	-0.36064	20.42094	326	44.732	0.1372

It has been observed that the ratio of the error to ensemble size should be minimum for the appropriate threshold value 20.0 m/s which is fulfils the above criterion of the summation of the square of the error of the observed value from the theoretical. So this can be taken as the appropriate threshold value for the cyclones occurred in Bay of Bengal.

The better fittings are obtained using the least square method in case of Bay of Bengal. The threshold value for the Bay of Bengal has been found to be 20.0 m/s (k = -0.15, s = 14.51 m/s).

Estimation of non-exceedance probability per year:

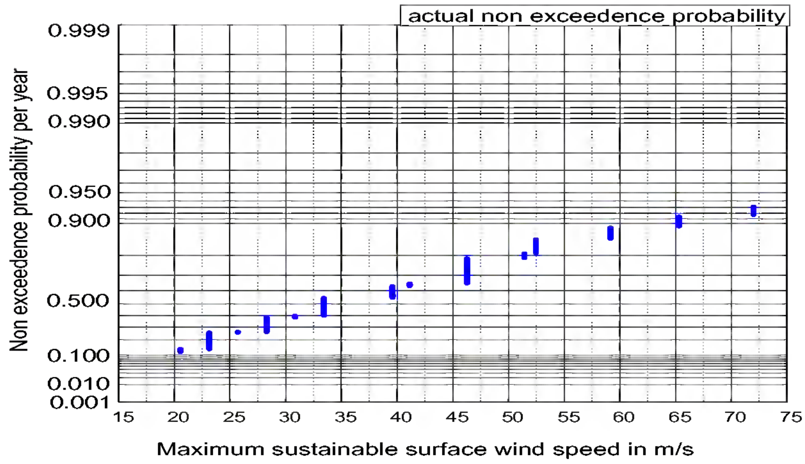

Fig. 4 Observed probability distribution of maximum sustainable surface wind speed in m/s

The generalised extreme value distribution has been obtained by the expressions (7) which have discussed above. According to appropriate threshold value (20.0 m/s) determined the observed probability of surface wind speed per year and then translated y values in Gumbel probability paper corresponding to calculate observed probabilities will be obtained. For this purpose the following expressions are used:

if p < 0.5 then

$$y = 5 - 5\left(\frac{-\ln(-\ln(1-p)) - fac005}{fac999 - fac005}\right) \quad (9)$$

if p > 0.5 then

$$y = 5 + 5\left(\frac{-\ln(-\ln p) - fac005}{fac999 - fac005}\right) \quad (10)$$

where

$$fac\,005 = -\ln(-\ln 0.005)$$

$$fac999 = -\ln(-\ln 0.999)$$

A curve on those obtained observed probability values vs surface wind speed has been plotted in Gumbel probability paper which is shown in figure 5.

Comparison between theoretical & observed non exceedance probability per year for zero curvature parameters:

Type 1 distribution for τ = 0

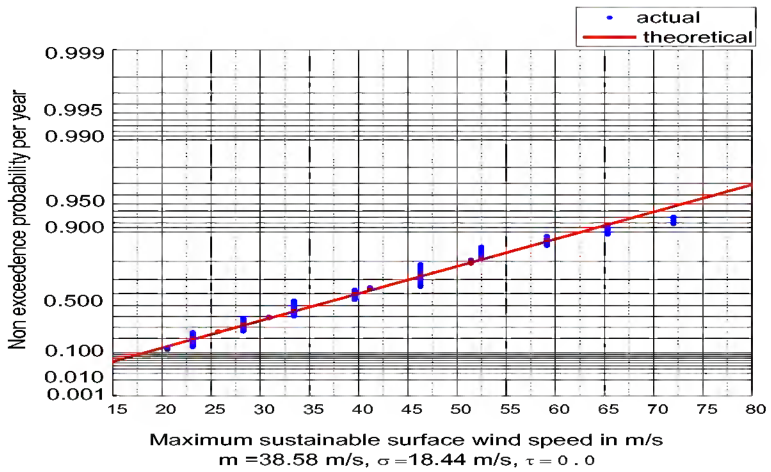

Fig. 5 Theoretical & actual non-exceedance probability per year Vs Gust wind speed. Comparison between theoretical & observed non-exceedence probability per year for non-zero curvature parameters

Type II or III distribution for τ ≠ 0

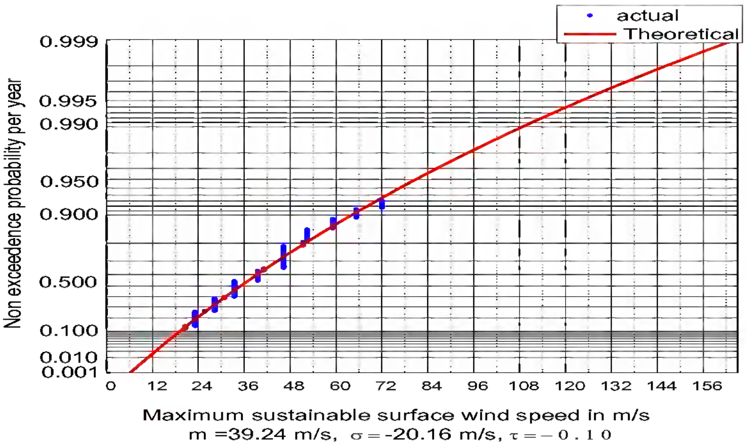

Fig. 6 Theoretical & actual non exceedance probability per year Vs Gust wind speed.

Table 3: All calculated values of f_1, f_2 m, σ, a & b corresponding to curvature parameters from-0.50 to 0.50 have been mentioned in the given below table

Curvature Parameters	Gamma function values		Parameters		Calculated variables (By Least Sq. Meth.)		Summa-tion of squared errors
	f_1	F_2	m	σ	a	b	
-0.50	1.77	Infinity	-	-	-	-	-
-0.45	1.61	2.62	43.28	-60.77	0.043	-0.25	20.78
-0.40	1.48	1.54	42.41	-41.79	0.036	-0.07	16.77
-0.35	1.38	1.03	41.68	-33.45	0.030	0.093	13.82
-0.30	1.29	0.73	41.06	-28.58	0.025	0.249	11.68
-0.25	1.22	0.52	40.52	-25.37	0.020	0.394	10.18
-0.20	1.16	0.36	40.04	-23.10	0.015	0.530	9.20
-0.15	1.11	0.24	39.62	-21.43	0.011	0.657	8.67
-0.10	1.06	0.14	39.24	-20.16	0.007	0.778	8.57
-0.05	1.03	0.06	38.90	-19.19	0.003	0.891	8.89
0.00	1.00	0.00	38.58	18.44	0. 069	-2.107	9.68
0.05	0.97	0.06	38.30	17.85	-0.003	1.102	11.02
0.10	0.95	0.11	38.03	17.40	-0.006	1.201	13.10
0.15	0.93	0.16	37.78	17.07	-0.009	1.296	16.23
0.20	0.91	0.21	37.54	16.83	-0.012	1.387	20.95
0.25	0.90	0.25	37.31	16.67	-0.015	1.475	28.37
0.30	0.89	0.29	37.09	16.58	-0.017	1.561	40.88
0.35	0.89	0.33	36.88	16.55	-0.020	1.645	65.01
0.40	0.88	0.37	36.68	16.58	-0.022	1.727	128.86
0.45	0.88	0.42	36.48	16.66	-0.025	1.807	
0.50	0.88	0.46	36.28	16.78	-0.027	1.887	

CONCLUSIONS

Cyclones are the major storms in India. Every year the structures in the coastal region especially those which are in the east coast of the India are affected by cyclone. From cyclone modeling it is observed that that the gust wind speed due to cyclone are best fitted by Fŕechet distribution (type-II Extreme value distribution with curvature parameters -0.10). The low value of curvature parameters clearly indicates that cyclone is strong storms. From this best fitted distribution the design wind speed can be estimated for the projected working life of a structure so that the structural damage will not occur due to cyclone.

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